Dynamic Buckling of a Rectangular Orthotropic Plate

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The elastic buckling of a simply-supported rectangular orthotropic plate, with initial imperfections, under a rapidly applied compressive load is analyzed. The large deflection plate equations are used to study inertial effects in the postbuckling phase. Numerical results for a controlled rate of load application show that the critical load is increased over the corresponding static case. The load-deflection relation is oscillatory in the postbuckling phase, thereby increasing the stresses in the plate over those in the static case. Initial imperfections decrease the critical load as well as increase the frequency and decrease the amplitude of the oscillations.

Nomenclature

a, b	= dimensions of plate, in.
c	= loading rate, psi/sec
D_x, D_y, D_1, D_{xy}	= flexural rigidities, in.-lb
E_x, E_y	= elastic moduli, psi
f_o	= amplitude of initial deflection, in.
$f(t)$	= amplitude of total deflection, in.
$F(x, y)$	= stress function, lb
g	= gravitational acceleration, in./sec^2
G_{xy}	= shear modulus, psi
h	= plate thickness, in.
m, n	= number of half waves
p	= average value of applied compressive stress, psi
p_{cr}	= static critical stress for orthotropic plate, psi
p_1	= static critical stress for square isotropic plate, psi
S	= dynamic similarity number
t	= time, sec
u, v	= in-plane deflection components, in.
U, V	= relative in-plane edge displacements, in.
$w_o(x, y)$	= initial deflection in normal direction, in.
$w(x, y, t)$	= total deflection in normal direction, in.
x, y, z	= rectangular coordinates
β	= aspect ratio
γ	= specific weight of plate, lb/in.^{3}
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	= middle surface strains
ζ_o	= nondimensional initial deflection
ζ	= nondimensional total deflection
ν, ν_{xy}, ν_{yx}	= Poisson's ratio
$\sigma_x, \sigma_y, \tau_{xy}$	= middle surface stresses, psi
τ	= nondimensional time
ω_{mn}	= natural frequency of unloaded plate, rad/sec

Introduction

THE study of the dynamic stability of structural elements first interested analysts about 40 years ago. In two survey articles[7,8] Hoff traced the development of the various problems and made classifications according to the method of loading. Several definitions of the term "dynamic stability" have been proposed, but the one adopted here will be that of Hoff in Ref. 8, "...any stability problem analyzed with the aid of Newton's equations of motion, or by any equivalent method, will be considered a dynamic stability problem."

One particular type of dynamic loading of practical interest is that applied by a rigid testing machine where the load is increased rapidly at a controlled rate. This type of loading has been used in the analysis of columns,[8] isotropic plates with damping,[6] and isotropic shells.[1] A rapidly applied load differs from an impact load in that the time required to reach the critical load is greater than the time required for a pressure wave to travel from one end of the element to the other. Therefore, for a rapidly applied load, inertial effects in the plane of the element are negligible and only the motion normal to the middle surface needs to be considered.

In recent years, the use of anisotropic materials—both homogeneous and composite—has increased. Many applications involve thin plates and shells under loadings which cause failure by buckling. The static stability of orthotropic plates and shells has been analyzed by numerous investigators and representative results are contained in Refs. 5, 9–11. Reference 3 is an extensive survey of the work which has been done in all areas of plate stability.

It is the purpose of this paper to study the elastic stability of a simply-supported rectangular orthotropic plate under a rapidly applied compressive load. Since the postbuckling behavior is to be considered, the large deflection plate equations must be used. The effects of initial imperfections are also included, because the critical loads are imperfection sensitive.

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Development of Equations

Consider a simply-supported rectangular orthotropic plate loaded as shown in Fig. 1. Assume that the natural axes of the material coincide with the coordinate axes so that the in-plane stress-strain relations may be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{1}{1-\nu_{xy}\nu_{yx}} \begin{bmatrix} E_x & \nu_{xy}E_y & 0 \\ \nu_{yx}E_y & E_y & 0 \\ 0 & 0 & (1-\nu_{xy}\nu_{yx})G_{xy} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{1}$$

and

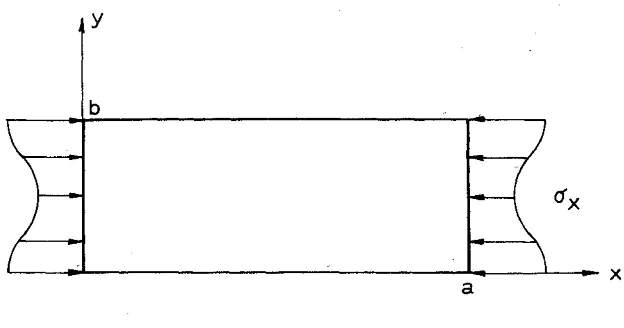

Fig. 1 Loaded rectangular plate.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1/E_x & -\gamma_{yx}/E_y & 0 \\ -\gamma_{xy}/E_x & 1/E_y & 0 \\ 0 & 0 & 1/G_{xy} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{2}$$

Four of the elastic constants are connected by the reciprocal relation

$$E_x/E_y = v_{xy}/v_{yx} \tag{3}$$

The critical load for plates is known to be sensitive to initial imperfections in the plate. The effect of these imperfections will be introduced through the second-order strain-displacement relations

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 - \dfrac{1}{2}\left(\dfrac{\partial w_o}{\partial x}\right)^2 \\ \varepsilon_y = \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 - \dfrac{1}{2}\left(\dfrac{\partial w_o}{\partial y}\right)^2 \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} - \dfrac{\partial w_o}{\partial x}\dfrac{\partial w_o}{\partial y} \end{cases} \tag{4}$$

In Eq. (4), $w(x, y, t)$ is the total displacement normal to the middle surface and $w_o(x, y)$ is the initial displacement normal to the middle surface. Also, u and v are the in-plane displacements in the x and y directions, respectively.

Using Eq. (4), and including the effect of inertia normal to the middle surface, the large deflection plate equations may be derived as outlined in Ref. 9. The compatibility condition is

$$\frac{1}{Ey}\frac{\partial^4 F}{\partial x^4} + 2\left(-\frac{v_{yx}}{E_y} + \frac{1}{2G_{xy}}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{1}{E_x}\frac{\partial^4 F}{\partial y^4} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w_o}{\partial x \partial y}\right)^2 + \frac{\partial^2 w_o}{\partial x^2}\frac{\partial^2 w_o}{\partial y^2} \tag{5}$$

and the equation of motion is

$$D_x \frac{\partial^4 (w - w_o)}{\partial x^4} + 2(D_1 + 2D_{xy})\frac{\partial^4 (w - w_o)}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 (w - w_o)}{\partial y^4} = h\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2} - \frac{\gamma}{g}\frac{\partial^2 w}{\partial t^2}\right] \tag{6}$$

In Eqs. (5) and (6), $F(x, y, t)$ is the stress function, h the plate thickness, γ the specific weight of the plate, and g the gravitational acceleration. The flexural rigidities are given by

$$D_x = \frac{E_x h^3}{12(1 - v_{xy} v_{yx})}, \qquad D_1 = \frac{E_x v_{yx} h^3}{12(1 - v_{xy} v_{yx})} = \frac{E_y v_{xy} h^3}{12(1 - v_{xy} v_{yx})}$$

$$D_y = \frac{E_y h^3}{12(1 - v_{xy} v_{yx})}, \qquad D_{xy} = \frac{G_{xy} h^3}{12}$$

The edges of the plate are initially straight and if all the edges remain straight after buckling, then the boundary conditions for both w and w_o are

$$\left.\begin{aligned} w = w_o &= 0 \\ \partial^2 w/\partial x^2 = \partial^2 w_o/\partial x^2 &= 0 \end{aligned}\right\} \text{ at } x = 0 \text{ and } x = a$$
$$\left.\begin{aligned} w = w_o &= 0 \\ \partial^2 w/\partial y^2 = \partial^2 w_o/\partial y^2 &= 0 \end{aligned}\right\} \text{ at } y = 0 \text{ and } y = b \tag{7}$$

If p is defined as the average value of the compressive stress in the x-direction and if there is no restraint in the y-direction at the edges $y = 0$ and $y = b$, then F must satisfy

$$\frac{1}{b}\int_o^b \sigma_x \, dy = \frac{1}{b}\int_o^b \frac{\partial^2 F}{\partial y^2} dy = -p, \quad \text{at } x = 0 \text{ and } x = a$$
$$\frac{1}{a}\int_o^a \sigma_y \, dx = \frac{1}{a}\int_o^a \frac{\partial^2 F}{\partial x^2} dx = 0, \quad \text{at } y = 0 \text{ and } y = b \tag{8}$$

The deflection function for the plate will be assumed to be the single mode

$$w(x, y, t) = f(t)\sin(m\pi x/a)\sin(n\pi y/b) \tag{9}$$

where $f(t)$ is the time-varying amplitude of w, and m and n are the number of half waves in the x and y directions, respectively. This form satisfies the boundary conditions, Eq. (7), and it will be shown later, Eqs. (23) and (24), that the edges remain straight after buckling. Budiansky[2] has suggested that it is reasonable to use the static buckling pattern to describe the dynamic response when the eigenvalues of the static problem are well separated. In analyzing the effects of initial imperfections, the initial and final shapes are usually assumed to be of the same basic form.[1,3,11] Thus, the initial shape of the plate will be taken as

$$w_o(x, y) = f_o \sin(m\pi x/a)\sin(n\pi y/b) \tag{10}$$

where f_o is the amplitude. This also satisfies the boundary conditions.

Substitute Eqs. (9) and (10) in the compatibility condition, Eq. (5), to obtain a differential equation relating the stress function to the assumed deflection functions.

$$\frac{1}{Ey}\frac{\partial^4 F}{\partial x^4} + 2\left(-\frac{v_{yx}}{E_y} + \frac{1}{2G_{xy}}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{1}{E_x}\frac{\partial^4 F}{\partial y^4} = \frac{m^2 n^2 \pi^4}{2a^2 b^2}\left[\cos\frac{2m\pi x}{a} + \cos\frac{2n\pi y}{b}\right][f^2 - f_o^2] \tag{11}$$

A solution of Eq. (11) which satisfies the conditions given by Eq. (8) is

$$F(x, y, t) = \frac{a^2 n^2 E_y}{32 b^2 m^2}(f^2 - f_o^2)\cos\frac{2m\pi x}{a} + \frac{b^2 m^2 E_x}{32 a^2 n^2}(f^2 - f_o^2)\cos\frac{2n\pi y}{b} - \frac{1}{2}py^2 \tag{12}$$

Although only two of the four independent material constants appear in Eq. (12), the other two will not be ignored, since all four will appear in the differential equation defining $f(t)$. The results of this analysis when specialized to an isotropic plate agree with those developed directly from the isotropic plate equations.[6] Next, substitute Eqs. (9, 10, and 12) in the equation of motion, Eq. (6), to determine the deflection amplitude as a function of time. The result of this substitution is

$$\frac{\pi^4}{h}\left[D_x\frac{m^4}{a^4} + 2(D_1 + 2D_{xy})\frac{m^2 n^2}{a^2 b^2} + D_y\frac{n^4}{b^4}\right][f - f_o] = \frac{m^2\pi^2}{a^2}pf - \frac{\gamma}{g}\frac{d^2 f}{dt^2} + \frac{\pi^4}{8}\left[\frac{m^4}{a^4}E_x\cos\frac{2n\pi y}{b} + \frac{n^4}{b^4}E_y\cos\frac{2m\pi x}{a}\right][f^2 - f_o^2]f \tag{13}$$

Since the coefficients in Eq. (13) involve functions of x and y, only an approximate solution can be found. This may be done by first determining average values of the coefficients. The Galerkin method, a weighted residual method, will be used. Each term in Eq. (13) is multiplied by $\sin(m\pi x/a)\sin(n\pi y/b)\,dx\,dy$ and integrated over the middle surface of the plate. The result is

$$\frac{d^2 f}{dt^2} + \frac{g\pi^4}{\gamma h}\left[D_x\frac{m^4}{a^4} + 2(D_1 + 2D_{xy})\frac{m^2 n^2}{a^2 b^2} + D_y\frac{n^4}{b^4}\right][f - f_o] - \frac{m^2\pi^2 gp}{a^2\gamma}f + \frac{g\pi^4}{24\gamma}\left[\frac{m^4}{a^4}E_x + \frac{n^4}{b^4}E_y\right][f^2 - f_o^2]f = 0 \tag{14}$$

where m and n are both odd integers.

Equation (14) is a nonlinear, second-order ordinary differential equation which relates the deflection amplitude, $f(t)$, to the imperfection amplitude, f_o, and to the average value, p, of the compressive stress. The time dependence of p is unspecified in Eq. (14) and, in order to solve the equation, the method of loading must be defined. One typical condition is a controlled rate of load application, which may be expressed as $p = p(t)$. Another is a controlled rate of relative edge displacement in the direction of loading, which may be expressed as $U = U(t)$. In solving Eq. (14) it will be assumed, as in the static case, that $n = 1$ is associated with the least critical load.

It will be assumed that the plate is at rest before loading so that the initial conditions for $f(t)$ are

$$\left.\begin{aligned} f &= f_o \\ df/dt &= 0 \end{aligned}\right\} \text{ at } t = 0 \tag{15}$$

Considering only initially perfect plates, $f_o = 0$, results for related problems may be derived from Eq. (14). By omitting the d^2f/dt^2 and f^3 terms, the resulting equation

$$\frac{\pi^2}{h}\left[D_x\frac{m^4}{a^4} + 2(D_1+2D_{xy})\frac{m^2n^2}{a^2b^2} + D_y\frac{n^4}{b^4}\right] - \frac{m^2}{a^2}p = 0 \quad (16)$$

is the characteristic equation for the corresponding static case using linear theory.[9] The critical load, with $n = 1$, is

$$p_{cr} = \frac{\pi^2 D_x}{b^2 h}\left[\left(\frac{mb}{a}\right)^2 + 2\frac{D_1+2D_{xy}}{D_x} + \frac{D_y}{D_x}\left(\frac{a}{mb}\right)^2\right]$$

and the least critical load

$$(p_{cr})_{min} = (2\pi^2/b^2h)[(D_x D_y)^{1/2} + D_1 + 2D_{xy}]$$

occurs when $(a/b)(D_y/D_x)^{1/2}$ is an integer.

The large deflection equation describing the static postbuckling behavior is obtained by omitting the d^2f/dt^2 term from Eq. (14).

$$\left\{\frac{\pi^2}{h}\left[D_x\frac{m^4}{a^4} + 2(D_1+2D_{xy})\frac{m^2n^2}{a^2b^2} + D_y\frac{n^4}{b^4}\right] - \frac{m^2}{a^2}p\right\}f + \frac{\pi^2}{24}\left[\frac{m^4}{a^4}E_x + \frac{n^4}{b^4}E_y\right]f^3 = 0 \quad (17)$$

Equation (17) shows that the deflection is zero until the critical load given by Eq. (16) is reached. After that point, nonzero deflections are possible and the load-deflection relation is a second-order parabola.

Finally, the square of the natural frequency of the unloaded plate in any symmetric mode, using linear analysis, is given by the coefficient of f in Eq. (14) when $p = 0$.

$$\omega_{mn}^2 = \frac{g\pi^4}{\gamma h}\left[D_x\frac{m^4}{a^4} + 2(D_1+2D_{xy})\frac{m^2n^2}{a^2b^2} + D_y\frac{n^4}{b^4}\right] \quad (18)$$

As an example of the solution of Eq. (14), consider the case where the average compressive stress increases linearly with time; $p = ct$. The solution will be somewhat more general if the following parameters are introduced. All are nondimensional except for p_1

$$p_1 = \frac{4D_x\pi^2}{a^2h} \qquad \tau = \frac{p}{p_1} = \frac{ct}{p_1}$$
$$\zeta = \frac{f}{h} \qquad R_{12} = \frac{D_1+2D_{xy}}{D_x}$$
$$\zeta_0 = \frac{f_0}{h} \qquad R_{22} = \frac{D_y}{D_x} = \frac{E_y}{E_x} \quad (19)$$
$$\beta = \frac{a}{b} \qquad S = \frac{p_1^3 g\pi^2}{c^2a^2\gamma}$$

With these substitutions, and for $n = 1$, Eq. (14) becomes

$$\frac{d^2\zeta}{d\tau^2} + S\left\{\frac{(m^4+2R_{12}m^2\beta^2+R_{22}\beta^4)}{4}(\zeta-\zeta_0) + \frac{(1-\nu_{xy}\nu_{yx})}{8}(m^4+R_{22}\beta^4)(\zeta^2-\zeta_0^2)\zeta - m^2\tau\zeta\right\} = 0 \quad (20)$$

The choice of the parameters p_1, R_{12}, and R_{22} was partly for convenience in comparing the results of the present analysis with those for isotropic plates. The parameter p_1 is the static critical stress for a simply-supported isotropic square plate of side a and rigidity D_x. For isotropic materials $R_{12} = R_{22} = 1$. The solution of Eq. (20) will be discussed later.

The expression in brackets in Eq. (20) is the load-deflection relation for the static case and is the nondimensional form of Eq. (17). The nondimensional parameter S is a dynamic similarity number. That is, all plates having the same aspect ratio, β, the same relative imperfection amplitudes, ζ_o, the same number of half waves, m, and the same rigidity ratios, R_{12} and R_{22}, will be dynamically equivalent if they have the same dynamic similarity numbers. It should also be noted that if the $Sm^2\tau\zeta$ term is omitted from Eq. (20) and if $\zeta_o = 0$, the resulting equation has the same form as the one describing the free oscillations of a mass and hardening spring system. The equation corresponding to Eq. (20) for an isotropic plate, when $R_{12} = R_{22} = 1$ and $\nu_{xy} = \nu_{yx} = \nu$, is

$$\frac{d^2\zeta}{d\tau^2} + S\left[\frac{(m^2+\beta^2)^2}{4}(\zeta-\zeta_0) + \frac{1-\nu^2}{8}(m^4+\beta^4)(\zeta^2-\zeta_0^2)\zeta - m^2\tau\zeta\right] = 0 \quad (21)$$

Edge Effects

After $f(t)$ has been found from Eq. (20), the membrane stresses may be determined from the stress function, Eq. (12), as

$$\sigma_x = -\frac{m^2\pi^2E_x}{8a^2}(f^2-f_0^2)\cos\frac{2n\pi y}{b} - p$$
$$\sigma_y = -\frac{n^2\pi^2E_y}{8b^2}(f^2-f_0^2)\cos\frac{2m\pi x}{a} \quad (22)$$
$$\tau_{xy} = 0$$

The relative edge displacement, U, in the x-direction may be defined as

$$U = \int_0^a \frac{\partial u}{\partial x}dx$$

The integrand, $\partial u/\partial x$, may be found by eliminating ε_x from the following strain-displacement and stress-strain relations

$$\varepsilon_x = \partial u/\partial x + \tfrac{1}{2}(\partial w/\partial x)^2 - \tfrac{1}{2}(\partial w_0/\partial x)^2$$
$$\varepsilon_x = \sigma_x/E_x - \nu_{xy}\sigma_y/E_y$$

Using Eqs. (3, 10, and 12) this gives

$$\frac{\partial u}{\partial x} = -\pi^2\left[\frac{m^2}{8a^2}\left(1+\cos\frac{2m\pi x}{a} - \cos\frac{2m\pi x}{a}\cos\frac{2n\pi y}{b}\right) - \nu_{yx}\frac{n^2}{8b^2}\cos\frac{2m\pi x}{a}\right](f^2-f_0^2) - \frac{p}{E_x}$$

and, when the integration is carried out,

$$U = -(m^2\pi^2/8a)(f^2-f_0^2) - pa/E_x \quad (23)$$

A similar analysis for V, the relative edge displacement in the y-direction, gives

$$V = -(n^2\pi^2/8b)(f^2-f_0^2) + \nu_{xy}pb/E_y \quad (24)$$

Equations (23) and (24) show that all the edges remain straight during loading, since both U and V are independent of x and y. The fact that the loaded edges remain straight while σ_x varies with y implies that the load is applied through a rigid member.

Example

To illustrate the solution of Eq. (20) numerical calculations were made for boron/epoxy plates of various geometries and at several loading rates. The material properties used[4] are shown in Table 1.

Representative results will be presented in this section for a plate 10 in. × 10 in. × 0.05 in. having a static buckling stress of 1075 psi with $m = 1$. The values of the rigidities and the parameters used in the solution of Eq. (20) are given in Table 2.

Equation (20) was solved numerically using a variable step fourth-order Runge-Kutta method. Three values of ζ_o—0.001, 0.01, and 0.01—and two values of S—82 and 328—were used. The

Table 1 Material properties

E_x	$= 40 \times 10^6$ psi
E_y	$= 4 \times 10^6$ psi
G_{xy}	$= 1.5 \times 10^6$ psi
ν_{xy}	$= 0.25$
ν_{yx}	$= 0.025$
γ	$= 0.0585$ lb/in.^{3}

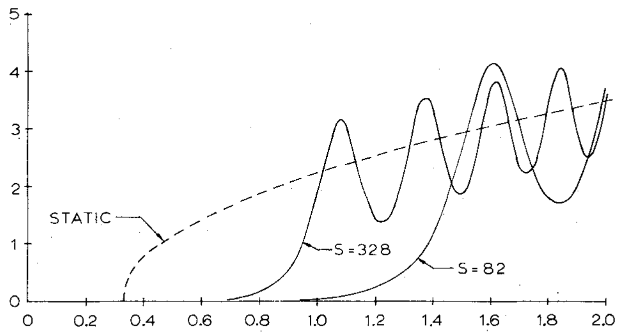

Fig. 2 Response curves for $\zeta_o = 0.001$.

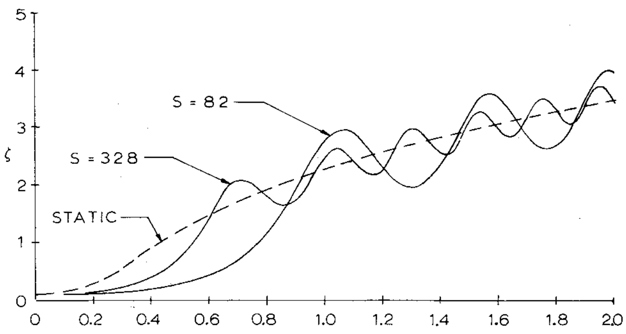

Fig. 4 Response curves for $\zeta_o = 0.1$.

Table 2 Rigidities and parameters used in example

$D_x = 419.3$ psi	a)	$c = 537500$ psi/sec
$D_y = 41.93$ psi		$S = 82$
$D_1 = 10.48$ psi		
$D_{xy} = 15.62$ psi		
$R_{12} = 0.09953$	b)	$c = 268750$ psi/sec
$R_{22} = 0.1$		$S = 328$
$p_1 = 3311$ psi		
$p_{cr} = 1075$ psi		
$m = 1$		
$\beta = 1$		

initial conditions for the problem are $\zeta = \zeta_o$ and $d\zeta/d\tau = 0$ at $\tau = 0$. The form of Eq. (20) is such that the solution for a perfect plate ($\zeta_o = 0$), using these initial conditions, is $\zeta \equiv 0$. Therefore, the case $\zeta_o = 0.001$ was used to approximate a perfect plate. The numerical solutions of Eq. (20) are plotted in Figs. 2–6 with the static solution included for comparison. Each shows that there is initially a relatively slow increase of deflection, then a rapid increase, and finally a series of nonlinear oscillations.

Figures 2–4 show the effect of the two loading rates on the response of plates having the same initial imperfections. In each case the response curve for the faster loading rate ($S = 82$) is to the right of that for the slower ($S = 328$). An examination of Fig. 2 shows that there is no definite point of instability as in static analyses of perfect plates. Rather, there is a region of instability where the slope of the ζ vs τ curve increases rapidly. For $S = 328$ this region is between $\tau = 0.7$ and $\tau = 0.9$, while for $S = 82$ it is between $\tau = 1.1$ and $\tau = 1.3$. Both response curves oscillate about the static response curve with increasing frequency. At any value of τ, the frequency for $S = 82$ is less than that for $S = 328$. The amplitude of the oscillations, relative to the static curve, is greater for the faster loading rate than for the slower. These oscillations result from the release of part of the strain energy stored in the plate before buckling when the deflection is zero or very small. The time to reach the region of instability is less for the slower rate and hence less strain energy is stored before buckling.

Oscillations have the effect of increasing the membrane and the bending stresses above the static stresses. Using values from Fig. 2 at $\tau = 1.6$ and for $S = 82$ the ratio of the maximum values of the membrane stresses σ_x(dynamic)/σ_x(static) is 1.6. These observations are valid for the curves in Fig. 3 ($\zeta_o = 0.01$) and Fig. 4 ($\zeta_o = 0.1$).

Figures 5 and 6 show the effect of variations in the amplitude of the initial imperfections when the loading rate is fixed. Both show that the region of instability occurs at larger values of τ as ζ_o decreases and also show that the slope of the ζ vs τ curve in the region of instability is more pronounced as ζ_o decreases. It should be noted that the results of this study apply only to linear elastic materials. It is possible that in the oscillatory phase the material may no longer be elastic. However, locating the regions of instability is the primary concern in a stability analysis and in these regions the material is still elastic.

Summary

The results of this investigation may be summarized as: a) A plate loaded rapidly will buckle at a higher critical stress than a plate loaded very slowly (statically). b) In the postbuckling phase

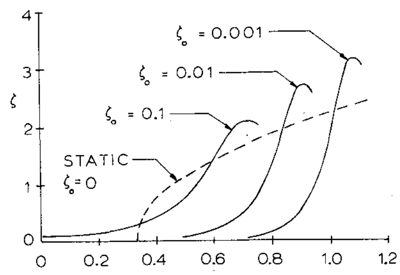

Fig. 5 Response curves for $S = 328$.

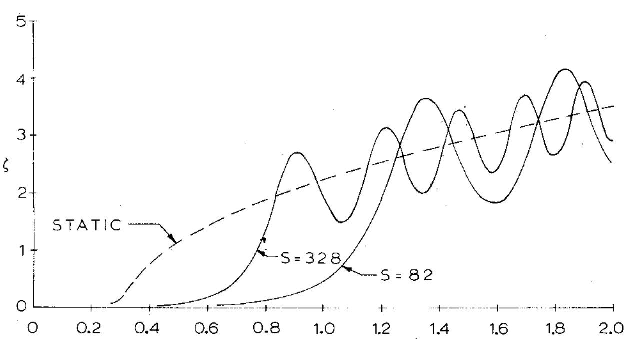

Fig. 3 Response curves for $\zeta_o = 0.01$.

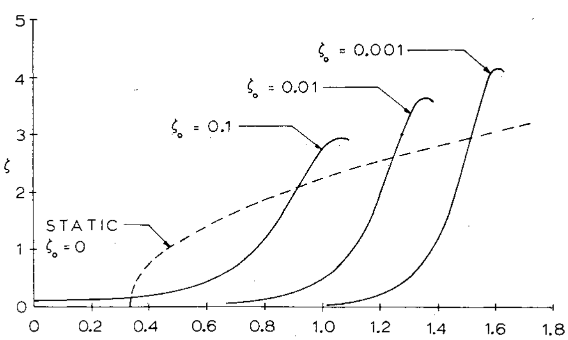

Fig. 6 Response curves for $S = 82$.

the rapidly loaded plate oscillates about the static load-deflection relation. This leads to higher stresses than in the static case. c) Initial imperfections cause a decrease in the critical stress. They also decrease the amplitude of the postbuckling oscillations. d) The amplitude of the oscillations, for a fixed ζ_o, increases as the loading rate is increased. e) The frequency of the oscillations, for a fixed loading rate, increases with τ and increases as ζ_o is increased.

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